# Length of a curve

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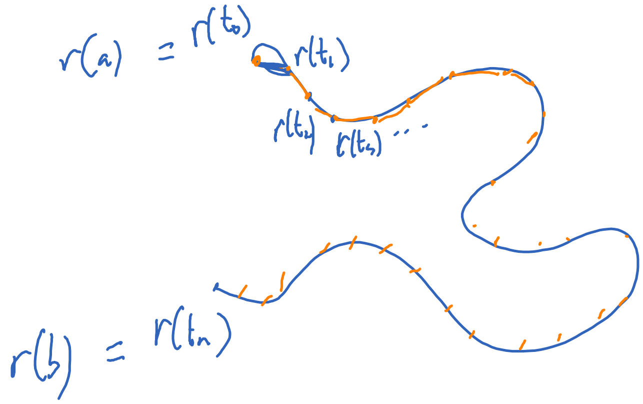
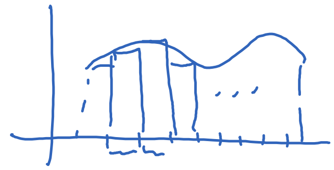



$r(t)$, $\underline{a \le t \le b}$

$L = ?$



$$L \approx \underbrace{|r(t_1) - r(t_0)|}_{\approx |r'(t_0)|\Delta t} + \underbrace{|r(t_2) - r(t_1)|}_{\approx |r'(t_1)|\Delta t} + \ldots$$

$$r'(t_0) = \lim_{t \to t_0} \frac{r(t) - r(t_0)}{t - t_0}$$

$t_1 \approx t_0$ :

$$r'(t_0) \approx \frac{r(t_1) - r(t_0)}{t_1 - t_0} \quad \Delta t$$

$$r(t_1) - r(t_0) \approx r'(t_0)\Delta t$$

$$\boxed{|r(t_1) - r(t_0)| \approx |r'(t_0)|\,\Delta t}$$

$$L \approx \underbrace{|r'(t_0)|\Delta t + |r'(t_1)|\Delta t + \ldots}$$

$$\approx \int_a^b |r'(t)|\,dt$$

$$L = \int_a^b |r'(t)|\,dt.$$

Ex:

$$r(t) = \langle \cos t, \sin t, t \rangle \qquad 0 \le t \le 2.$$

$$r'(t) = \langle -\sin t, \cos t, 1 \rangle$$

$$|r'(t)| = \sqrt{(\sin t)^2 + (\cos t)^2 + 1^2} = \sqrt{2}$$

$$L = \int_0^2 \sqrt{2}\,dt = 2\sqrt{2}.$$